Optimal Polynomial Approximation of Photovoltaic Panel Characteristics Using a Stochastic Approach

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Abstract:

The essence of this article is finding the optimal degree of the \( p = f(v) \) real photovoltaic panel characteristics approximation polynomial. The characteristics are considered as one realization of a stochastic system and it is the result of long-term measuring. The outputs are the coefficients of an approximation polynomial of an optimal degree. For its calculation, we use the well-known Euclidean norm of residues. The advantage of this approach is that it takes all the influences on the panel’s attributes into consideration (solar irradiation, temperature, aging, random effects). It is necessary for the approximation to carry out a rotation of the measured characteristics and a backwards rotation of the approximation polynomial course. This method enables us to create a mathematical or numerical model of a real photovoltaic panel of any type. All the algorithms and experiments were done using MATLAB® system.

Keywords:

Photovoltaic panel, modelling, stochastic approach, MATLAB

1. Introduction

The issue of the measuring and modelling of real photovoltaic panels (hereinafter referred to as PV panels) and systems draws a lot of attention at recent time. This fact is proved by the large number of published articles. Among them, we can trace two basic approaches to models. The first approach is based on the idealized Shockley’s equation which models a basic PV cell. The modelling of the whole panels and systems is then carried out with corrections to real attributes, e.g. nonlinear parametric loss resistances, temperature dependence etc. These models formed the vast majority until recent time, see [1-4].

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New techniques that use different approaches for the modelling of real PV systems have been emerging recently. The main reason is growing complexity of PV power plants in practice, production of new heterogeneous materials and the need of a fast, robust and accurate simulation [5]. Modelling and control of real photovoltaic devices are often published and behavioural approach is also mentioned [6, 7]. Identified models are implemented in various environments, e.g. in Simulink® [8].

This led to the emergence of adaptive and pseudo-random approaches and models for simulations in the real time [9, 10]. Attempts to analytic procedures and neural networks are used for the modelling [11-13]. There are also works that deal with the robustness of models, or combine technical and economic aspects in the process of modelling [13, 14].

Fig. 1 System for PV panel automatic measurement

Nowadays, the measuring of real PV panels and systems is a standardized process. Fig. 1 shows the fundamental scheme for low power systems.

Fig. 2 Record of string inverter input and its rotation for better approximation

The panel is connected to a professional string inverter and then to a load which can be either a distribution network or a variable resistor. In practice, the basic
operating point is situated near the maximum power point (Maximum Power Point Tracker – MPPT method) [7]. This statement is the basic idea of the article. Fig. 2 shows a typical result of a long term measuring. Apparently, these characteristics are not a deterministic dependence. It is a typical realization of a non-stationary stochastic process. Further phases of model processing take this assumption into consideration.

2. Basic Idea of Approximation

The main objective of this method is to carry out the optimal polynomial approximation of \( p = f(v) \) characteristic from Fig. 2. The procedure of the approximation consists of two crucial steps:

- finding the optimal polynomial degree and
- finding the coefficients of the optimal polynomial.

A large number of experiments shows that polynomials are suitable approximation functions for real measured \( p = f(v) \) characteristics. However, there is one fundamental problem that has to be solved. The first measured characteristic showed in Fig. 2 cannot be approximated by any ordinary function due to its ambiguity. That is why the measured record must be rotated; see the second course in Fig. 2.

![Block diagram of polynomial approximation process](image)

*Fig. 3 Block diagram of polynomial approximation process*
This serious problem was resolved by the following procedure:
- rotation of the characteristics 90 degrees to the right;
- actual process of approximation (main algorithm objective);
- backward rotation of the polynomial course.

The whole process is showed in the block diagram in Fig. 3.

To determine the optimal degree of the approximation polynomial, the Euclidean norm of residues is used [15]. This norm is defined as follows:

\[
\Delta_E = \sqrt{\sum_{k=1}^{n} (p_k - f_k)^2},
\]

where \( p_f \) is the course of the approximation polynomial, \( f \) is the course (vector) of the original approximated function, \( n \) is the length of the measured vector and \( k \) is the sequential index of the current input signal sample. The \( f \) function, in this case, is the measured \( p = f(v) \) characteristic. The \( \Delta_E \) unit is a unit of the approximated function.

The Euclidean norm is well known and used cumulative error between two vectors.

For the actual process of the polynomial approximation, these internal functions of MATLAB system [16] were used:
- \textit{polyfit} – finding the coefficients of the given degree;
- \textit{polyval} – calculation of the approximation course.

When the optimal polynomial is calculated the following final form can be written:

\[
v(P) = a_mP^m + a_{m-1}P^{m-1} + \ldots + a_1P + a_0,
\]

where \( v \) and \( P \) denote voltage and power according to the description below. Calculation of the optimal polynomial coefficients \( a_m, a_{m-1}, \ldots, a_1, a_0 \) is the main process objective.

3. Practical Example

We use the long-term data record from Fig. 2. According to the procedure showed in Fig. 3, the rotation of the measured characteristic is carried out 90 degrees right. In accordance with the block scheme in Fig. 3, the dependence of the \( \Delta_E \) on the degree of the approximation polynomial is drawn, see Fig. 4.

The polynomial of the 5\textsuperscript{th} degree, where the \( \Delta_E \) value is not substantially decreasing, appears to be optimal. Some techniques for the optimal degree evaluation are presented below.

The specified characteristic is therefore approximated by the polynomial of the 5\textsuperscript{th} degree. The system determines its coefficients, see Fig. 5. The vector of the coefficients showed by Fig. 5 is listed in descending order with the power of the independent variable. Number of decimal places is rounded to four places. The output numerical format \textit{short e} is used in the MATLAB environment.

After the optimal approximation, the backward rotation of the resulting polynomial course is automatically carried out. The resulting graph is drawn with white colour because of lucidity. It is interesting that the polynomial approximation can be also evaluated for many hundreds or thousands of measured points.
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![Image](image_url)

**Fig. 4** Dependence of the $\Delta E$ on the approximation polynomial degree of the rotated input measured signal

![Image](image_url)

**Fig. 5** Optimal approximation and optimal polynomial coefficients list

Optimal coefficients: $a_5 = 9.0250\times10^{-13}$, $a_4 = 2.7780\times10^{-9}$, $a_3 = -3.1892\times10^{-6}$, $a_2 = 1.6741\times10^{-3}$, $a_1 = -4.0380\times10^{-1}$, $a_0 = 1.8997\times10^2$

Fig. 6 shows the source text of the application in MATLAB for the calculation of the norm of residues $\Delta E$. There are two interesting internal commands. The command `polyfit` calculates the polynomial coefficients vector for the defined polynomial degree $k$. The command `polyval` calculates the approximation curve $f$ with defined polynomial $p$. The vector $f$ is used for $\Delta E$ calculation according to (1). The source code in Fig. 6 is simple and standard. The used formatting commands are not described more.

In Fig. 4 the optimal polynomial degree of the approximation process is highlighted. There are a few techniques how to evaluate the optimum numerically. In case of monotonously decreasing courses a normalized norm $\Delta_E = \frac{\Delta}{\Delta_{E_{\text{max}}}}$ is calculated. In our concrete example: $\Delta_{E_{\text{max}}} = 138.1105V$. The course of the normalized error is shown in Fig. 7 (upper graph). To define the optimal polynomial degree the relative error of two adjacent points must be calculated, see lower graph in Fig. 7. The optimal degree 5 has the relative error $\delta_{E_{N5}} = 1.8813\%$. This value is less than user-defined limit $\delta_{E_{\text{user}}} = 5\%$. That is why the application evaluates the polynomial degree 5 as the optimum. The use-defined limit depends on concrete approximated physical system and users’ experience.

--- Euclidean norm of residues ---

```matlab
subplot(2,1,2) % create axes in tiled positions
axis([1 10 50 400]) % axis scaling
format short g % output data format
DeltaE=zeros(10,2);
for k=1:10 % polynomial degrees
    p=polyfit(power,voltage,k);
    f=polyval(p,power);
    DeltaE(k,1)=k; % norm
    DeltaE(k,2)=sqrt(sum((f-voltage).^2));
    hold on
    plot(k,DeltaE(k,2),'ko','markersize',8,'linewidth',2)
end
```

Fig. 6 Calculation of the Euclidean Norm of Residues $\Delta_E$ in MATLAB

Fig. 7 Normalized norm and relative error for automatic optimum evaluation
5. Numerical Conditionality

If the resulting approximation polynomial changes the values of coefficients throughout the course it is necessary to calculate the numerical conditionality of the polynomial defined as [15]:

$$N_C = \frac{\Delta R_p}{\Delta C_p},$$ (3)

where $\Delta C_p$ is the change of polynomial coefficient, $\Delta R_p$ is the change of its root, and $N_C$ is so-called polynomial conditionality number. In Fig. 8 there is a concrete course of vector $N_c$ compared with $\Delta E$. There is the logarithmic scale on the vertical axis.

![Approximation quality evaluation](image)

**Fig. 8 Course of the numerical conditionality number $N_C$**

The actual process of the $N_C$ calculation can be programmed using the original algorithm, see Fig. 9. It is a function with one input parameter $p$ (optimal polynomial coefficients vector). On the basis of mathematical theory the task is well-conditioned for the condition $N_C \leq 1$. In practice the condition can be friendlier $N_C \leq 100$.

```matlab
function [ki,j,Cp]=polynomial_condition(p)
    n=length(p)-1; % polynomial degree
    k=roots(p); % polynomial roots
    k=sort(k); % roots sorting
    q=polyder(p); % polynomial derivative
    for i=1:n
        for r=1:(n+1)
            kij(i,r)=((k(i)^r-2))/polyval(q,k(i)).*p(n+2-r);
        end
    end
    Nc=abs(max(max(kij))); % conditionality number
```

**Fig. 9 Calculation of the polynomial conditionality number $N_C$**
As presented above our example is a relatively well-conditioned numerical task, because $N_C = 6.31$.

6. Conclusion

This article describes the original method of optimal polynomial approximation of long-term measuring of $p = f(v)$ characteristics of a real photovoltaic panel. The approximated characteristics are considered as a realization of a non-stationary stochastic process. The calculation of the optimal degree of the approximation polynomial uses the Euclidean norm of residues which is a well-known criterion in practice. The actual approximation process is based on the rotation of measured characteristics and the internal functions of MATLAB system. The advantage of the described method is the fact that it includes all the influences on the photovoltaic panel. As a brief note, the chapters on the optimal polynomial degree evaluation and numerical conditionality of the polynomial are presented. The described technique can be used to solve various technical problems with characteristics which are approximated with difficulty.

References


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