Effect of the Accuracy of Target Range Measurement on the Accuracy of Fire

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Abstract:
The article deals with the mutual relation between the accuracy of the target range measurement and the accuracy of fire. The stereoscopic target range measurement and its errors are briefly described in the first part. The second part of the article is focused on the impact of inaccurate target range measurement on projectile’s vertical position of the point of impact. Various ballistic systems are used for the simulation of the effect. Results of simulations are discussed from two points of view. First point of view is the use of the results for the selection of the most suitable ballistic system for the firing on the target at given range. The second one offers utilisation of the results for the setting of requirements on the accuracy of target range measurement for given ballistic system and size of the target.

Keywords:
Exterior ballistics, passive range finder, target range measurement, trajectory modelling

1. Introduction
The fire accuracy of the guns is largely affected by the accuracy of the determination of the relative position between the weapon and the target. In case of the direct fire, the most important characteristic of the target position with respect to the weapon is the slant range as the basic input factor for the fire data computation [1].

Nowadays, the laser rangefinder is the most frequently used device for the target slant range measurement for its promptness, accuracy and reliability. Nevertheless, the recent research [2, 3] shows the relatively high rate of the laser measurement errors due to the atmosphere effects, false reflections, laser beam divergence, and others. From the tactical point of view, the most important disadvantage of the laser
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rangefinder is the fact that the laser rangefinder during its operation emits a relatively large amount of energy, which is detectable by the enemy’s reconnaissance means [1]. Especially in the situations when it is necessary to measure the target range several times (repeating of the measurement because of the measurement fault, effort to determine the direction and velocity of the target motion by means of the several measured points of the target trajectory and so on) the enemy gets sufficient time and information for taking the appropriate countermeasures, or the counterattack.

One possible solution of this problem is to utilize the methods of the digital photogrammetry when the target position and the slant range are determined from at least two images of the target which were synchronously acquired from the different camera positions. The fundamental advantage of this approach is that the CCD or CMOS detectors of cameras do not emit such an amount of energy which could be detected by the enemy, and only the radiation emitted or reflected by the target is passively received. Furthermore, these types of detectors could operate in the relatively wide range of the electromagnetic spectra, so they can be used for target range measurement also under reduced visibility and in the night.

2. Passive Target Range Measurement

The determination of the target position based on the image information processing results from the fundamental relations between the target coordinates with respect to the global coordinate system \( X, Y, Z \) connected with the Earth and the image coordinates of the target with respect to the local coordinate system \( y_s, z_s \), connected with the camera detector. This relation can be expressed by the formulas in following form [4, 5]

\[
\begin{align*}
    z_s &= z_0 + c \frac{r_{11} (X - X_0) + r_{21} (Y - Y_0) + r_{31} (Z - Z_0)}{r_{13} (X - X_0) + r_{23} (Y - Y_0) + r_{33} (Z - Z_0)}, \\
    y_s &= y_0 + c \frac{r_{12} (X - X_0) + r_{22} (Y - Y_0) + r_{32} (Z - Z_0)}{r_{13} (X - X_0) + r_{23} (Y - Y_0) + r_{33} (Z - Z_0)},
\end{align*}
\]

(1)

where:

- \( y_s, z_s \) are the object image coordinates with respect to the local coordinate system connected with the camera detector,
- \( y_0, z_0 \) are the coordinates of the principal point of the camera with respect to the local coordinate system connected with the camera detector,
- \( c \) is the principal distance of the camera,
- \( X_0, Y_0, Z_0 \) are the coordinates of the projection centre of the camera with respect to the global coordinate system,
- \( X, Y, Z \) are the coordinates of the target with respect to the global coordinate system and
- \( r_{ij} \) are the elements of the rotational transformation matrix of the camera with respect to the global coordinate system.

The slant range \( L \) of the target can be determined using the following formula

\[
L = \sqrt{(X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2}.
\]

(2)

The principal point coordinates and the camera constant are the elements of the inner camera orientation. These elements are determined during camera calibration
and they are considered to be the constants for the measurement process. The projection centre coordinates and the rotation of the camera with respect to the global coordinate system are the elements of the outer camera orientation and they can vary during the measurement process in general. However, in the scope of the slant range determination the relative elements of the outer camera orientation are substantial, i.e. the knowledge of the relative position of the reference camera with respect to the measuring camera is more important. The elements of the relative outer orientation of the camera are also determined during camera calibration and they also remain constant.

In the case of incorrect determination of the inner and relative outer camera orientation elements, systematic error is introduced into the measurement process. This error can be eliminated by re-calibration of the camera, though.

Random error in the slant range measurement process is caused by errors in the determination of the image coordinates of the target. Hereinafter the effect of the incorrect image coordinates determination on the slant range computation and consequently on the target hit error will be discussed.

It is obvious from the formulas (1) that the image coordinates of the target can be determined from one image. But for the inverse operation, when the global coordinates of the target shall be reconstructed for the known image coordinates, one image is insufficient, because we have only two conditional equations for three unknown coordinates \( X, Y, Z \). Therefore, at least two images have to be acquired for obtaining four conditional equations, which can be used for deriving the equation system with the unique solution. In the case of using two cameras the term camera base is used. The camera base \( b \) is the element of the relative outer camera orientation in principle and it can be expressed by formula

\[
b = \sqrt{(X_{01} - X_{00})^2 + (Y_{01} - Y_{00})^2 + (Z_{01} - Z_{00})^2},
\]

where:

- \( X_{00}, Y_{00}, \) and \( Z_{00} \) are the coordinates of the reference camera projection centre with respect to the global coordinate system and
- \( X_{01}, Y_{01}, \) and \( Z_{01} \) are the coordinates of the measuring camera projection centre with respect to the global coordinate system.

For better understanding, the following conditions have been adopted:

- the optical axes of the both cameras are parallel to each other and perpendicular to the base of the cameras,
- the beginning of the global coordinate system is identical with the projection centre of the reference camera,
- the optical axis of the reference camera is coincidental with the longitudinal axis \( X \) of the global coordinate system and it goes through the point which represents the target,
- the axes \( y \) and \( z \) of the local coordinate system connected with the camera detector are parallel to the corresponding axes \( Y \) and \( Z \) of the global coordinate system and
- for the measuring camera it is valid that \( X_{01} = X_{00}, Y_{01} = Y_{00} \) and \( Z_{01} = b \).

This normal case of stereo restitution is shown in Fig. 1. The individual symbols in this figure will be explained in this article hereafter. On the basis of these preliminary conditions we can transform the relation between the target global coordinates and image coordinates into the simplified form
\[ z_{s1} = z_{01} + c_1 \frac{b}{L}, \]  

where:

- \( z_{s1} \) is the horizontal image coordinate of the target for the measuring camera,
- \( z_{01} \) is the horizontal coordinate of the principal point of the measuring camera,
- \( c_1 \) is the principal distance of the measuring camera,
- \( b \) is the base of the cameras and
- \( L \) is the slant range of the target with respect to the projection centre of the reference camera.

From equation (4) the resulting formula for the slant range computation can be derived in form

\[ L = b \frac{c_1}{z_{s1} - z_{01}}. \]  

**Fig. 1** Normal case of the stereo restitution

### 3. Errors of the Passive Target Range Measurement

In the case of using the digital cameras for acquiring the images, the fundamental error in the image coordinates determination is caused by the spatial discretization of the continuous brightness distribution in the plane of the camera detector. It means that the
image coordinate $z_{s1}$ does not take the values from the continuous interval $\langle 0; z_m \rangle$, where $z_m$ is the physical width of the camera detector but from the vector of discrete values $Z$ whose elements can be computed by means of the formula

$$z_i = \Delta z i$$ for $i = 0..n_m - 1$, \hspace{1cm} (6)

where:

- $\Delta z$ is the minimal distinguishable spatial increment (or spatial resolution) of the camera detector and
- $n_m$ is the number of spatial increments in the particular direction.

In practice the $\Delta z$ is frequently equal to the dimension of the camera detector element (pixel).

If the determined value of the discrete image coordinate is $z_{s1}$, for this value following formula is valid

$$z_{s1} = \Delta z n_{z1},$$ \hspace{1cm} (7)

where:

- $n_{z1}$ is the number of the image pixel, which was estimated as the representation of the target.

In general, due to the quantization noise mentioned above, the nominal value of image coordinate $z_{s1}$ will differ from $z_{s1}$ and it can take its value from the interval

$$z_{s1} \in \langle \Delta z (n_{z1} - 1); \Delta z (n_{z1} + 1) \rangle.$$ \hspace{1cm} (8)

To determine the error in the slant range measurement it is necessary to examine the two limiting values of the image coordinate from the interval above.

Using the substitution

$$z_{s1} - z_{01} = \Delta z n_{p1},$$ \hspace{1cm} (9)

where $n_{p1} = n_{z1} - z_{01}/\Delta z$, the equation (5) can be written in form

$$\bar{L} = \frac{bc_1}{\Delta z n_{p1}}.$$ \hspace{1cm} (10)

The slant range error $\Delta L$ is given by the difference between the evaluated slant ranges for limiting values of the discrete image coordinates according to the formula

$$\Delta L = L_{s1} - L_{s1} = \frac{bc_1}{\Delta z (n_{p1} - 1)} - \frac{bc_1}{\Delta z (n_{p1} + 1)} =$$

$$= \frac{bc_1}{\Delta z (n_{p1} - 1)} \left( \frac{1}{n_{p1} - 1} - \frac{1}{n_{p1} + 1} \right) = \frac{bc_1}{\Delta z} \left( \frac{n_{p1} + 1 - n_{p1} + 1}{n_{p1}^2 - 1} \right) =$$

$$= \frac{bc_1}{\Delta z} \left( \frac{2}{n_{p1}^2 - 1} \right).$$ \hspace{1cm} (11)

Using the formula (10), it can be written for $n_{p1}$

$$n_{p1} = \frac{bc_1}{\Delta z \bar{L}}.$$ \hspace{1cm} (12)
Substituting (12) into (11), the new formula for the determination of the slant range error will be obtained

\[
\Delta L = b \frac{c_1}{\Delta z} \frac{2}{b^2 c_1^2} = bc_1 \frac{2\Delta z^2 \bar{E}^2}{\Delta z^2 b^2 c_1^2 - \Delta z^2 \bar{E}^2} = \frac{2bc_1\Delta z^2 \bar{E}^2}{b^2 c_1^2 - \Delta z^2 \bar{E}^2}.
\] (13)

For relatively high resolutions of camera detectors \((\Delta z < 10^{-5} \text{ m})\) and relatively short target ranges \((L < \text{ca. } 3000 \text{ m})\) it can be written

\[\Delta z^2 \bar{E} \to 0,\] (14)

therefore, the formula (13) can be transformed into form

\[\Delta L = \frac{2\Delta z \bar{E}}{bc_1}.
\] (15)

It means that absolute error of the measured slant range is indirectly proportional to the base of cameras and principal distance of the camera and directly proportional to the resolution of the camera detector and the square of the measured distance. Finally, the relative error of the slant range measurement can be expressed by equation in form

\[\frac{\Delta L}{L} = \frac{2\Delta z \bar{E}}{bc_1}.
\] (16)

The estimated error of the range measurement can be used for determination of the point of impact shift with respect to the aiming point.

Example: \(\Delta z = 6.45 \mu\text{m}, b = 2 \text{ m}, c_1 = 0.072 \text{ m},\) target range \(L\) varies from 300 to 1200 m. The estimated errors are shown in the Tab. 1.

**Tab. 1 Estimated relative error of the measured target distance**

<table>
<thead>
<tr>
<th>Nominal target distance (L) [m]</th>
<th>300</th>
<th>600</th>
<th>900</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image coordinates (z_s - z_{01}) [mm]</td>
<td>0.480</td>
<td>0.240</td>
<td>0.160</td>
<td>0.120</td>
</tr>
<tr>
<td>Discretized image coordinates (\Delta z_{n,1}) [mm]</td>
<td>0.477</td>
<td>0.239</td>
<td>0.161</td>
<td>0.123</td>
</tr>
<tr>
<td>Discretized target distance (\bar{L}) [m]</td>
<td>301.70</td>
<td>603.39</td>
<td>893.02</td>
<td>1175.03</td>
</tr>
<tr>
<td>Relative difference between (L) and (\bar{L}) [%]</td>
<td>0.56</td>
<td>0.56</td>
<td>-0.78</td>
<td>-2.08</td>
</tr>
<tr>
<td>Estimated error (\Delta L) [m]</td>
<td>8.15</td>
<td>32.61</td>
<td>71.44</td>
<td>123.69</td>
</tr>
<tr>
<td>Relative estimated error [%]</td>
<td>2.7</td>
<td>5.4</td>
<td>8.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

4. **Ballistic Considerations**

The current experimental firing task solved in the research project can be described as a standard NATO target of size 2.3 \(\times\) 2.3 m moving at distances up to 650 m with velocity about 5 m\(\cdot\)s\(^{-1}\).

At this moment the ballistic system of calibre 7.62\(\times\)54 mm for the purposes of the research project is utilised. The used ballistic system is capable to successfully
fulfil the current experimental firing task, but it is not suitable for the accurate firing at the longer ranges of fire which would be closer to the firing tasks in military practice, especially due to increasing steepness of the projectile trajectory at longer ranges of fire and also due to its sensitivity to atmospheric effects (especially cross wind).

For these reasons it is necessary to consider selection of a new ballistic system that will be capable to successfully hit the moving standard target at ranges of fire about 1000 m. The weapon system with this capability would be more suitable for the application into practice.

4.1. Exterior Ballistic Modelling

The question of exterior ballistic modelling can be divided into two parts. The first part is focused on the selection and use of a suitable trajectory model. The second part deals with the most efficient calculation of the aiming angles.

Trajectory Model

Among several available trajectory models the point mass trajectory model was chosen. This model is sufficiently accurate for the calculation of trajectories for the direct fire ballistic systems and also the input data of various ballistic systems are relatively easily available. The trajectories can be calculated very quickly in comparison with the more sophisticated trajectory models and the speed of calculation is a very important parameter in the ballistic computers.

This point mass trajectory model consists of six first order differential equations. These equations describe projectile motion as a motion of a point mass in the space.

\[
\frac{dv_y}{dt} = -c_{43} \left( \frac{p_N}{p_{0N}} \right) (v_x - w) \left( \frac{\tau_{0N}}{\tau_N} \right)^{0.5} G_{43},
\]

\[
\frac{dx}{dt} = v_x,
\]

\[
\frac{dv_y}{dt} = -c_{43} \left( \frac{p_N}{p_{0N}} \right) (v_x - w) \left( \frac{\tau_{0N}}{\tau_N} \right)^{0.5} G_{43} - g,
\]

\[
\frac{dv_y}{dt} = v_y,
\]

\[
\frac{dv_z}{dt} = -c_{43} \left( \frac{p_N}{p_{0N}} \right) (v_x - w) \left( \frac{\tau_{0N}}{\tau_N} \right)^{0.5} G_{43},
\]

\[
\frac{dz}{dt} = v_z,
\]

where:
- \(c_{43}\) is ballistic coefficient (drag law 1943),
- \(p_N\) is standard atmospheric pressure,
- \(p_{0N}\) is standard atmospheric pressure at sea level,
- \(w\) is wind speed,
- \(\tau_N\) is standard virtual air temperature,
- \(\tau_{0N}\) is standard virtual air temperature at sea level,
- \(G_{43}\) is drag function (drag law 1943).
Details of the exterior ballistic model can be found in the [6, 7, 8, 9].

The system of equations (17) is solved by means of standard Runge-Kutta method with variable time step.

**Calculation of Aiming Angles**

The basic function of any ballistic computer is to calculate the aiming angles (angle of elevation and bearing) for the particular firing task. To successfully fulfil this task it is necessary to know the following:

- trajectory model (in this case the point mass trajectory model),
- characteristics of weapon system (initial velocity, ballistic coefficient),
- atmospheric conditions (air pressure, temperature, direction and speed of wind),
- position of the target (slant range of the target, azimuth and angle of site).

The angle of elevation can be calculated for the particular firing task iteratively. In this case one of the most efficient methods, the secant method, was used. The method can be described by the following relation:

\[
\theta_0(i) = \theta_0(i-1) + \left[ 0 - y(i-1) \right] \frac{[\theta_0(i-1) - \theta_0(i-2)]}{[y(i-1) - y(i-2)]}
\]  

where:

- \(\theta_0\) is the angle of elevation,
- \(y\) is the height of point of impact on the target.

The iteration process is ended when the height of point of impact \(y\) differs from 0 (centre of the target) by less than 1 calibre, i.e. \(7.62 \times 10^{-3}\) m. All calculations are carried out under standard atmospheric conditions [10].

**4.2. Selection of Ballistic Systems**

For the purpose of this analysis, the most widespread and available ballistic systems from the calibre 5.56 up to calibre 30 mm were chosen. It is necessary to mention that the attention is primarily focused on the weapon systems of calibres from 12.7 up to 20 mm. Ballistic systems of calibre higher than 20 mm are too powerful for the existing experimental gun-carriage and there are also other practical problems (e.g. large danger areas in firing ranges, high cost of ammunition, …).

Among the small calibre ballistic systems were chosen following systems fielded in the Army of the Czech Republic (ACR) or other NATO armies:

- 5.56×45 mm (standard NATO),
- 7.62×39 mm (ACR),
- 7.62×51 mm (standard NATO),
- 7.62×54 mm (ACR),
- 12.7×99 mm (standard NATO),
- 12.7×108 mm (ACR),
- 14.5×114 mm (ACR).

Basic exterior ballistic properties of above mentioned small calibre ballistic systems are summarised in Tab. 2.
Effect of the Accuracy of Target Range Measurement on the Accuracy of Fire

Tab. 2 Exterior ballistic characteristics of selected small calibre ballistic systems

<table>
<thead>
<tr>
<th></th>
<th>5.56x4</th>
<th>7.62x3</th>
<th>7.62x5</th>
<th>7.62x5</th>
<th>12.7x9</th>
<th>12.7x1</th>
<th>14.5x1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0 [\text{m} \cdot \text{s}^{-1}]$</td>
<td>1006</td>
<td>743</td>
<td>856</td>
<td>855</td>
<td>945</td>
<td>850</td>
<td>890</td>
</tr>
<tr>
<td>$c_{45}$</td>
<td>10.50</td>
<td>10.30</td>
<td>6.70</td>
<td>6.57</td>
<td>3.60</td>
<td>3.60</td>
<td>3.50</td>
</tr>
<tr>
<td>$X_{\text{max}} [\text{m}]$</td>
<td>2853</td>
<td>2587</td>
<td>3833</td>
<td>3888</td>
<td>6345</td>
<td>6057</td>
<td>6309</td>
</tr>
</tbody>
</table>

Automatic cannons of following calibres were added rather due to the complexity of the selection:
- 20x102 mm (ACR),
- 30x165 mm (ACR),
- 30x211 mm (ACR).

Basic exterior ballistic properties of chosen automatic cannons are summarised in Tab. 3.

Tab. 3 Summary of exterior ballistic characteristics of selected automatic cannons

<table>
<thead>
<tr>
<th></th>
<th>20x102</th>
<th>30x165</th>
<th>30x211</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0 [\text{m} \cdot \text{s}^{-1}]$</td>
<td>1050</td>
<td>970</td>
<td>1000</td>
</tr>
<tr>
<td>$c_{45} [\text{kg} \cdot \text{m}^{-2}]$</td>
<td>4.55</td>
<td>2.05</td>
<td>2.00</td>
</tr>
<tr>
<td>$X_{\text{max}} [\text{m}]$</td>
<td>5551</td>
<td>9756</td>
<td>10110</td>
</tr>
</tbody>
</table>

5. Effect of Accuracy of the Target Range Measurement

The range of the target is one of the most important characteristics of the target for the calculation of the aiming angles. The accuracy of this characteristic directly affects the accuracy of fire and consequently also the target hit probability.

The effect of the accuracy of the target range measurement on the position of the point of impact is expressed by means of change of height of point of impact on the target. Calculation of aiming angles is realised for the measured target range (with error) and the projectile trajectory is calculated for the real (correct) target range and the height of point of impact is recorded. The height of point of impact should be considered in terms of the mean point of impact.

Using the formula (16) we can estimate that the range of errors of the target range measurement will not exceed ±15 % of the target range up to 3500 m (for the rangefinder configuration mentioned in the example in Chapter 3). The change of the height of point of impact is presented in dependency on the relative range of the target. The relative range of the target is defined as a ratio of both measured and real range of the target. Three real ranges of the target (600, 900, and 1200 m) were chosen for the calculations.

Results of these calculations are presented in the following figures and tables. The tables show the results with the 5 % step. The currently used weapon systems are marked dark grey and the perspective weapon systems are marked light grey.
Fig. 2 Effect of inaccuracy of target range measurement – 600 m

Tab. 4 Change of height of point of impact due to target range inaccuracy – 600 m

<table>
<thead>
<tr>
<th>Calibre</th>
<th>-15 %</th>
<th>-10 %</th>
<th>-5 %</th>
<th>0 %</th>
<th>+5 %</th>
<th>+10 %</th>
<th>+15 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.56×45 mm</td>
<td>-0.89</td>
<td>-0.62</td>
<td>-0.32</td>
<td>0.00</td>
<td>0.35</td>
<td>0.73</td>
<td>1.14</td>
</tr>
<tr>
<td>7.62×39 mm</td>
<td>-1.81</td>
<td>-1.25</td>
<td>-0.64</td>
<td>0.00</td>
<td>0.68</td>
<td>1.39</td>
<td>2.14</td>
</tr>
<tr>
<td>7.62×51 mm</td>
<td>-0.79</td>
<td>-0.54</td>
<td>-0.27</td>
<td>0.00</td>
<td>0.29</td>
<td>0.59</td>
<td>0.91</td>
</tr>
<tr>
<td>7.62×54 mm</td>
<td>-0.77</td>
<td>-0.53</td>
<td>-0.27</td>
<td>0.00</td>
<td>0.28</td>
<td>0.58</td>
<td>0.89</td>
</tr>
<tr>
<td>12.7×99 mm</td>
<td>-0.43</td>
<td>-0.29</td>
<td>-0.15</td>
<td>0.00</td>
<td>0.15</td>
<td>0.30</td>
<td>0.46</td>
</tr>
<tr>
<td>12.7×108 mm</td>
<td>-0.54</td>
<td>-0.36</td>
<td>-0.18</td>
<td>0.00</td>
<td>0.19</td>
<td>0.38</td>
<td>0.58</td>
</tr>
<tr>
<td>14.5×114 mm</td>
<td>-0.48</td>
<td>-0.33</td>
<td>-0.16</td>
<td>0.00</td>
<td>0.17</td>
<td>0.34</td>
<td>0.52</td>
</tr>
<tr>
<td>20×102 mm</td>
<td>-0.38</td>
<td>-0.26</td>
<td>-0.13</td>
<td>0.00</td>
<td>0.13</td>
<td>0.27</td>
<td>0.41</td>
</tr>
<tr>
<td>30×165 mm</td>
<td>-0.35</td>
<td>-0.23</td>
<td>-0.12</td>
<td>0.00</td>
<td>0.12</td>
<td>0.24</td>
<td>0.36</td>
</tr>
<tr>
<td>30×211 mm</td>
<td>-0.32</td>
<td>-0.22</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.11</td>
<td>0.22</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Fig. 3 Effect of inaccuracy of target range measurement – 900 m

Tab. 5 Change of height of point of impact due to target range inaccuracy – 900 m

<table>
<thead>
<tr>
<th>Calibre</th>
<th>−15 %</th>
<th>−10 %</th>
<th>−5 %</th>
<th>0 %</th>
<th>+5 %</th>
<th>+10 %</th>
<th>+15 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.56x45 mm</td>
<td>−4.04</td>
<td>−2.81</td>
<td>−1.47</td>
<td>0.00</td>
<td>1.58</td>
<td>3.29</td>
<td>5.12</td>
</tr>
<tr>
<td>7.62x39 mm</td>
<td>−6.40</td>
<td>−4.40</td>
<td>−2.26</td>
<td>0.00</td>
<td>2.40</td>
<td>4.95</td>
<td>7.64</td>
</tr>
<tr>
<td>7.62x51 mm</td>
<td>−2.85</td>
<td>−1.98</td>
<td>−1.03</td>
<td>0.00</td>
<td>1.11</td>
<td>2.30</td>
<td>3.57</td>
</tr>
<tr>
<td>7.62x54 mm</td>
<td>−2.78</td>
<td>−1.93</td>
<td>−1.00</td>
<td>0.00</td>
<td>1.08</td>
<td>2.24</td>
<td>3.47</td>
</tr>
<tr>
<td>12.7x99 mm</td>
<td>−1.19</td>
<td>−0.81</td>
<td>−0.41</td>
<td>0.00</td>
<td>0.42</td>
<td>0.87</td>
<td>1.32</td>
</tr>
<tr>
<td>12.7x108 mm</td>
<td>−1.51</td>
<td>−1.03</td>
<td>−0.52</td>
<td>0.00</td>
<td>0.54</td>
<td>1.11</td>
<td>1.69</td>
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<tr>
<td>14.5x114 mm</td>
<td>−1.34</td>
<td>−0.91</td>
<td>−0.46</td>
<td>0.00</td>
<td>0.48</td>
<td>0.97</td>
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</tr>
<tr>
<td>20x102 mm</td>
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<td>−0.75</td>
<td>−0.39</td>
<td>0.00</td>
<td>0.40</td>
<td>0.83</td>
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</tr>
<tr>
<td>30x165 mm</td>
<td>−0.86</td>
<td>−0.58</td>
<td>−0.29</td>
<td>0.00</td>
<td>0.30</td>
<td>0.60</td>
<td>0.91</td>
</tr>
<tr>
<td>30x211 mm</td>
<td>−0.80</td>
<td>−0.54</td>
<td>−0.27</td>
<td>0.00</td>
<td>0.28</td>
<td>0.56</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Fig. 4 Effect of inaccuracy of target range measurement – 1200 m

Tab. 6 Change of height of point of impact due to target range inaccuracy – 1200 m

<table>
<thead>
<tr>
<th>Calibre</th>
<th>Error of target range measurement [m]</th>
</tr>
</thead>
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<tr>
<td>5.56×45 mm</td>
<td>-11.30 -7.85 -4.09 0.00 4.44 9.25 14.45</td>
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<tr>
<td>7.62×39 mm</td>
<td>-16.24 -11.22 -5.82 0.00 6.26 12.99 20.24</td>
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<tr>
<td>7.62×51 mm</td>
<td>-7.71 -5.32 -2.75 0.00 2.94 6.07 9.39</td>
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<tr>
<td>7.62×54 mm</td>
<td>-7.52 -5.20 -2.69 0.00 2.87 5.93 9.17</td>
</tr>
<tr>
<td>12.7×99 mm</td>
<td>-2.64 -1.81 -0.93 0.00 0.97 2.00 3.08</td>
</tr>
<tr>
<td>12.7×108 mm</td>
<td>-3.42 -2.34 -1.20 0.00 1.26 2.60 4.02</td>
</tr>
<tr>
<td>14.5×114 mm</td>
<td>-2.97 -2.03 -1.04 0.00 1.09 2.25 3.47</td>
</tr>
<tr>
<td>20×102 mm</td>
<td>-2.62 -1.81 -0.93 0.00 1.00 2.06 3.21</td>
</tr>
<tr>
<td>30×165 mm</td>
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</tr>
<tr>
<td>30×211 mm</td>
<td>-1.59 -1.07 -0.54 0.00 0.56 1.13 1.71</td>
</tr>
</tbody>
</table>
6. Discussion

The obtained results can be utilised in two different ways:

- for selection of optimal ballistic system,
- for setting of requirements on the accuracy of target range measurement.

6.1. Selection of Optimal Ballistic System

In this first case the range of fire, accuracy of the target range measurement, and the size of the target are given and the optimal ballistic system is searched. A new firing task is defined as a standard NATO target of size 2.3 x 2.3 m moving at ranges up to 1200 m with velocity of about 5 m·s⁻¹. The aiming point is placed into the centre of the target.

The selection of the optimal ballistic system will be shown on the example of the 7.62×54 and the 12.7×99 systems. From Tables 4, 5, and 6 the heights of point of impact for all ranges of target and the ±5 % accuracy of measurement were separated and placed into the Fig. 5. The horizontal lines represent the bottom and upper edge of the target.

From Fig. 5 it can be clearly seen that the target range of 900 m is the limit for the currently used ballistic system 7.62×54; other effects (e.g. cross wind) are not taken into account for this assessment. On the other hand, the perspective ballistic system of calibre 12.7×99 can be used with the ±5 % accuracy of target range measurement against targets at distances up to 1200 m. It should be also mentioned that the marks in the figure represent the mean points of impact. The position of the mean point of impact at the target edge (upper or bottom) means that only 50 % of all projectiles fired will hit the target.

![Fig. 5 Comparison of current and perspective ballistic system](image-url)
6.2. Setting of Requirements on the Accuracy of Target Range Measurement

In this case the particular ballistic systems, range of fire and size of the target are given and the required accuracy of the target range measurement is searched for. In other words, the question is how accurately the target range must be measured for a given ballistic system and the size of the target.

The setting of requirements on the accuracy of the measurement can be shown on the example of the chosen ballistic systems and standard NATO target at the distance of 1200 m as shown in Fig. 5. To hit the standard NATO target at this distance with currently used ballistic system, it is necessary to measure the range of the target with better than $\pm 2\%$ accuracy. In case of using the 12.7×99 system the target range must be measured with better than $\pm 6\%$ accuracy. For the 30 mm ballistic systems it is sufficient yet the $\pm 9\%$ accuracy of measurement.

Fig. 6 Comparison of current and perspective ballistic system

Figures 2 and 3 present the required accuracy of the target range measurement for each ballistic system and given size of the target in a similar way.

7. Conclusion

The simple mathematical model of passive target range measurement is presented in this article. This model was used for deriving the relative range error. From the error analysis of the passive range measuring system, the minimal magnitudes of errors of the target range measurement were obtained. This analysis became the basis for the external ballistics assessment of the effect of the accuracy of the target range measurement on the accuracy of fire.
Effect of the Accuracy of Target Range Measurement on the Accuracy of Fire

From the ballistic analysis which has been conducted it can be concluded that from the investigated ballistic systems the 30 mm automatic cannons show the smallest change of the height of point impact. On the other hand, these ballistic systems are too powerful and therefore they are not suitable for mounting on the existing gun carriage. The size of the dangerous areas and cost of munition for these systems cannot be neglected either.

From the perspective group of ballistic systems (12.7 – 20 mm) the ballistic systems 12.7×99 and 20×102 are the most suitable ones. These ballistic systems pose a good and comparable insensitivity to the target range measurement accuracy but from the performance point of view the 12.7×99 system is more suitable than the 20×102 one.

The conclusion based on the analysis is that the ballistic system 12.7×99 should be used for the firing against targets at distances about 1000 m. The dispersion of fire of this ballistic system must be also investigated to get complete information for the final decision.

References

Acknowledgement
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